

**5 a** Prove that  $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, -\pi < x < \pi$  and hence show **6** M that  $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$ .

**b** Write Fourier series for the function  $f(x) = 2x - x^2$  in (0, 3) and hence deduce **6** M that  $\frac{\pi}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ 

## OR

6 a If  $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$  then show that  $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}.$ 6 M

**b** Obtain Fourier series for the function  $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$  and hence 6 M

deduce that 
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

## UNIT-IV

7 a Show that 
$$\int_{0}^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin x\lambda \, d\lambda = \begin{cases} \frac{\pi}{2}, & \text{if } 0 < x < \pi \\ 0, & \text{if } x > \pi \end{cases}$$
 making use of Fourier **6** M integral.

**b** Applying Fourier sine integral formula, show that

$$e^{-ax} - e^{-bx} = \frac{2(b^2 - a^2)}{\pi} \int_0^\infty \frac{\lambda \sin \lambda x}{\left(\lambda^2 + a^2\right) \left(\lambda^2 + b^2\right)} \, d\lambda \text{ where } a, b > 0.$$
**6** M
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Find the Fourier cosine transform of  $f(x) = \frac{1}{1 + x^2}$  and applying it find Fourier sine 8 12 M transform of  $\phi(x) = \frac{x}{1+x^2}$ .

**UNIT-V a** Form the partial differential equation by eliminating the arbitrary function from 9 6 M  $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ 

**b** Solve by method of separation of variables  $3u_x + 2u_y = 0$  and  $u(x, 0) = 4e^{-x}$ 6 M

## OR

10 A homogenous rod of conducting material of length 100cm has its ends kept at zero 12 M Temperature and the temperature initially  $u(x, 0) = \begin{cases} x; & 0 \le x \le 50 \\ 100 - x; & 50 \le x \le 100 \end{cases}$ . Find the temperature u(x, t) at any time.